YR 9 NAPLAN NUMERACY

Student Name:

....

The Year 9 NAPLAN Test Numeracy Workbook

2nd Edition

Contents

IMPORTANT NOTE

Checking the answers regularly is important as it ensures you are not continually making the same error. The answers can also give you clues on how to solve a problem if you are unsure about it.

REMEMBER

Always check with **B.O.B.**
(**B**ack **O**f **B**ook)

What you must know for NAPLAN

Times tables 3

Perimeter 115

Circumference of Circle	$C = 2\pi r$ or $C = \pi d$
Perimeter of a Square	$P = 4s$
Perimeter of a Rectangle	$P = 2l + 2w$
Other figures	*(Just add the sides)*

Area 115

Area of a Circle	$A = \pi r^2$
Area of a Square	$A = s^2$
Area of a Rectangle	$A = l \times w$
Area of a Parallelogram	$A = l \times h$
Area of a Triangle	$A = \frac{1}{2} \times \text{base} \times \text{height}$

Surface Area 125, 126

Surface Area of a Sphere	$SA = 4\pi r^2$
Surface Area of a Cube	$SA = 6s^2$
Surface Area of a Rectangular Prism	$SA = 2lh + 2lw + 2wh$
Surface Area of a Cylinder	$SA = 2\pi rh + 2\pi r^2$ (closed cylinder)
	$SA = 2\pi rh + \pi r^2$ (open end cylinder)
Surface Area of a Cone	$SA = \pi rS + \pi r^2$
Surface Area of a Square-Based Pyramid	$SA = 2\,sS + s^2$
Surface Area of a Triangular-Based Pyramid	$SA = \text{Area of base} + \frac{1}{2} PS$
Surface Area of a Triangular Prism	$SA = (s_1 + s_2 + s_3)\,d + s_1 h$

Volume 124–126

Volume of a Sphere	$V = \frac{4}{3}\pi r^3$
Volume of a Cube	$V = s^3$
Volume of a Rectangular Prism	$V = lwh$
Volume of a Cylinder	$V = \pi r^2 h$
For figures which have vertical sides	$V = \text{Area of the base} \times \text{height}$
Volume of a Cone	$V = \frac{1}{3}\pi r^2 h$
Volume of a Square-Based Pyramid	$V = \frac{1}{3} s^2 h$
Volume of a Triangular-Based Pyramid	$V = \frac{1}{3}(\frac{1}{2} b h H)$
Volume of a Triangular Prism	$V = \frac{1}{2} b h d$

Speed, Time and Distance 98

$$Speed = \frac{Distance}{Time}$$

$$Time = \frac{Distance}{Speed}$$

$$Distance = Speed \times Time$$

Types of Triangles 139

Scalene triangles are triangles with all three angles less than 90°.

Isosceles triangles have two equal angles. The sides opposite these angles are also equal.

Equilateral triangles have all three sides equal. All angles are also equal to 60°.

Angles from Parallel Lines 137

F Rule	Corresponding angles are equal in value.
U Rule	Co-interior angles add up to 180°.
Z Rule	Alternate angles are equal in value.

Statistics 108

Mean	Average score found by adding all the scores and dividing by number of scores.
Median	Middle number when scores in order.
Mode	Most common score (bimodal).
Range	Distance between the highest and lowest score.

Percentages 64, 65

Percentages of quantities	$x\% \text{ of } y = \frac{x}{100} \times y$
Percentage one quantity is of another	$\% = \frac{a}{b} \times 100$

Number Theory and Basic Operations

NUMBER THEORY

A. Natural Numbers

Prime Numbers

Prime numbers are numbers which have only 2 factors → itself and one.

EXAMPLE 1: $3, 7, 23$ are prime numbers.

Note: *1 is neither a prime nor a composite number, 2 is the only even prime number.*

Composite Numbers

Composite numbers are numbers which have more than 2 factors.

EXAMPLE 2: 24 has factors $1, 2, 3, 4, 6, 8, 12, 24$.

Exercise 1.1

1. In the table below, shade in all the prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Are all numbers that are not shaded composite numbers?

G. Index Notation

RULE 6: *Index notation (or powers) is a convenient way of expressing a product with the same factors.*

EXAMPLE 11:

a) Write 6^4 in expanded form.

b) Write $2 \times 3 \times 2 \times 2 \times 3 \times 5$ in index notation.

Solution:

a) $6 \times 6 \times 6 \times 6$

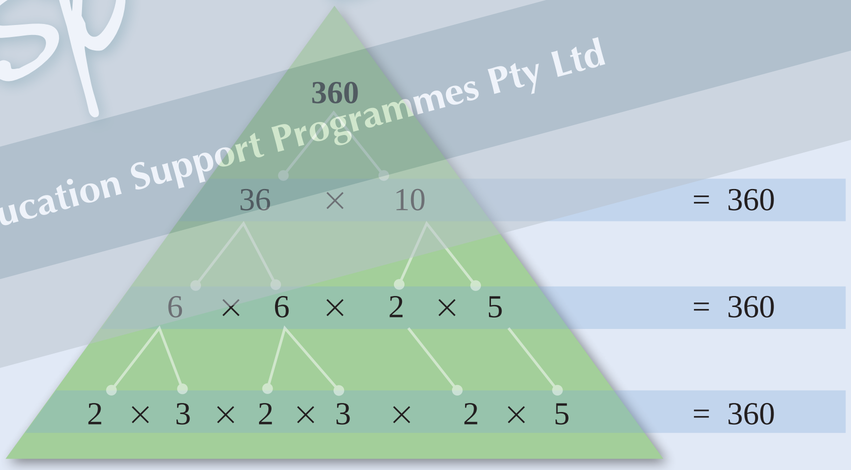
b) $2^3 \times 3^2 \times 5^1$

(*Note:* $5^1 = 5$)

All natural numbers can be written as a product of prime numbers. One way to do this is by using a factor tree as shown below.

EXAMPLE 12: Express 360 as a product of prime numbers.

Solution:



$\therefore\ 2^3 \times 3^2 \times 5 = 360.$

I. Indices

RULE 7: *Indices (plural of index) is another name for powers or exponents.*

The '6' is called the **coefficient**; it is the constant multiplied by a variable or expression.

$$6a^n$$

The 'n' is called the **index** or **exponent** or **power**.

The 'a' is called the **base**; any number or expression raised to a power is called a base.

Exercise 1.9

1. Answer the following questions.

a) $5a^3$	b) a	c) $6(5a)^2$
The coefficient is ______	The coefficient is ______	The coefficient is ______
The base is ______	The base is ______	The base is ______
The index is ______	The index is ______	The index is ______

INDEX LAWS

QUESTION	EXAMPLE or PROOF	LAW
$a^n \times a^m$	$a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a)$ $= a^7$ $or = a^{3+4}$	**INDEX LAW 1** $a^n \times a^m = a^{n+m}$
$a^n \div a^m$	$a^5 \div a^2 = \frac{(a \times a \times a \times a \times a)}{(a \times a)}$ $= a^3$ $or = a^{5-2}$	**INDEX LAW 2** $a^n \div a^m = a^{n-m}$
a^1	$4^4 = 256$ $4^3 = 64$ $4^2 = 16$ *As we reduce the power by 1 we divide our result by the base (i.e. 4).*	**INDEX LAW 3** $a^1 = a$

Exercise 1.10

1. Use index law 1 to simplify the following:

a) $6^4 \times 6^3$ b) $a^4 \times a^3$ c) $2^5 \times 2^8$ d) $x^4 \times x^{-2}$

2. Use index law 2 to simplify the following:

a) $6^4 \div 6^3$ b) $a^4 \div a^3$ c) $2^5 \div 2^8$ d) $x^4 \div x^{-2}$

3. Use index law 3 to simplify the following:

a) 4^1 b) 2^1 c) x^1 d) p^1

4. Use index law 4 to simplify the following:

a) 4^0 b) x^0 c) $2p^0$ d) $(2p)^0$

5. Use index law 5 to simplify the following:

a) 4^{-1} b) 2^{-1} c) x^{-1} d) p^{-1}

6. Use index law 6 to simplify the following:

a) 4^{-2} b) x^{-2} c) x^{-4} d) x^{-n}

7. Use index law 7 to simplify the following:

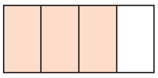
a) $(2p)^3$ b) $(3n)^4$ c) $(4s)^2$ d) $(pq)^n$

CHAPTER 2

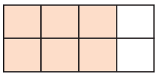
Fractions

A. Equivalent Fractions

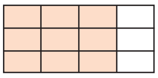
Consider the following representations of fractions:



→ $\frac{3}{4}$



→ $\frac{6}{8}$



→ $\frac{9}{12}$

You can see that $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

notice that $\frac{3\times 2}{4\times 2} = \frac{6}{8}$

and $\frac{3\times 3}{4\times 3} = \frac{9}{12}$

We say that $\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$ are equivalent fractions.

RULE 1: *By multiplying the numerator and denominator by the same number, an **equivalent fraction** is formed.*

EXAMPLE 1: Express each of the following with a denominator of 36.

a) $\frac{3}{4}$

b) $\frac{2}{3}$

Solution:

a) $\frac{3}{4} = \frac{\boxed{}}{36}$

since $4\times 9 = 36$

$\therefore \frac{3\times 9}{4\times 9} = \frac{\boxed{27}}{36}$

b) $\frac{2}{3} = \frac{\boxed{}}{36}$

since $3\times 12 = 36$

$\therefore \frac{2\times 12}{3\times 12} = \frac{\boxed{24}}{36}$

EXAMPLE 2: Express the following in their simplest form.

a) $\frac{24}{32}$

b) $\frac{84}{100}$

Solution:

a) The Highest Common Factor (HCF) of 24 and 32 is 8.

$$\therefore \frac{24 \div 8}{32 \div 8} = \frac{3}{4}$$

b) The HCF of 84 and 100 is 4.

$$\therefore \frac{84 \div 4}{100 \div 4} = \frac{21}{25}$$

EXAMPLE 3: Arrange the following fractions in order of size (smallest to largest).

a) $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$

Solution:

a) The Lowest Common Multiple (LCM) of 6, 4 and 3 is 12.

$$\therefore \frac{5}{6} = \frac{10}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

$$\textit{and} \quad \frac{2}{3} = \frac{8}{12}$$

Thus, the order of size is $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.

Exercise 2.1

1. Express the following with a denominator of 24.

a) $\frac{1}{2}$ b) $\frac{5}{6}$ c) $\frac{3}{8}$ d) $\frac{7}{12}$

2. Express the following as hundredths.

a) $\frac{3}{10}$ b) $\frac{1}{4}$ c) $\frac{4}{5}$ d) $\frac{13}{25}$

Decimal Fractions

RULE 1: *Decimal numbers (or decimal fractions) are numbers which come after the decimal place.*

The decimal place divides the whole numbers and the fractions, thus:

$$32.45 \;=\; \underbrace{32}_{\textit{whole number}} + \underbrace{\frac{4}{10} + \frac{5}{100}}_{\textit{fractions}}$$

A. Place Value

Just as 30 → 3 × 10 + 0 units

then 0.04 → 0 units + 0 tenths + 4 hundredths

EXAMPLE 1: Express 3.504 in expanded form.

Solution:

$$\underset{\uparrow}{3.504} \;=\; 3 + \frac{5}{10} + \frac{4}{1000}$$

(omit the zero column)

EXAMPLE 2: What is the value of the 6 in the number 324.0461?

Solution:

$$324.\underset{\textit{tenths}}{0}\,\underset{\textit{hundredths}}{4}\,\underset{\textit{thousandths}}{6}\,1$$

$\therefore$ 6 *has the value of* $\frac{6}{1000}$

HANDY DECIMAL/FRACTION CONVERSIONS

See page 62 for a table combined with percentage equivalents.

G. Rounding Off Decimals

Consider $\frac{25}{9}$. When converting to a fraction divide 25 by 9:

$$\begin{array}{r} 2.777\\ 9\,\overline{)\,25.000}\\ \underline{18}\\ 7.0\\ \underline{6.3}\\ .70\\ \underline{63}\\ 70\\ \underline{63} \end{array} \quad \rightarrow \quad \textit{and so on . . .}$$

Thus, 25 ÷ 9 = 2.777. . .

we call this 2.7 recurring and we write it as $2.\dot{7}$.

EXAMPLE 11: Find the value of 38 ÷ 7 correct to two decimal places.

Solution:

$$\begin{array}{r} 5.428\\ 7\,\overline{)\,38.000}\\ \underline{35}\\ 3.0\\ \underline{2.8}\\ .20\\ \underline{14}\\ 60\\ \underline{56}\\ 4 \end{array}$$

Divide to 3 decimal places, then round off to 2 decimal places.

It is often useful to use your calculator for these calculations.

$\therefore$ 5.428 $\doteqdot$ 5.43 to 2 decimal places.

Exercise 3.6

1. Write:

   a) 0.7341 correct to two decimal places

   b) 37.074 correct to one decimal place

2. Find a decimal approximation for:

   a) $7\frac{2}{3}$ b) $\frac{23}{7}$ c) $\frac{35}{9}$

(Complete question 3 using your calculator.)

3. Use your calculator to find the following. (Round off your answers to 2 decimal places.)

   a) 23.71×6.04 b) $(3.71)^2 \times 6.4$

   c) $0.7 \div 1.7$ d) $\frac{9}{17}$

IMPORTANT NOTE

When using a calculator to evaluate an expression like

$$\frac{3.17 + 4.62}{37.1 + 46.8}$$

it is important to remember to use brackets to link the top and bottom lines.

Thus, $\frac{3.17 + 4.62}{37.1 + 46.8}$ becomes $(3.17 + 4.62) \div (37.1 + 46.8)$

Using the $\boxed{\text{EXP}}$ key on your calculator

EXAMPLE 13: ***Use your calculator to find the answer to*** $3.21 \times 10^7 \times 4.2 \times 10^{-3}$.

Solution: Key in 3.21 $\boxed{\text{EXP}}$ 7 × 4.2 $\boxed{\text{EXP}}$ – 3 $\boxed{=}$

The result is 134 820.

NOTE: ***Some calculators use*** $\boxed{\times 10^x}$ ***rather than*** $\boxed{\text{EXP}}$.

Exercise 3.9

(Complete using your calculator.)

1. Use your calculator to find:

a) $32\,000 \times 7243 \div 0.006$ b) $0.007 \times 0.067 \times 0.0003$

c) $467 \times 362.4 \times 8426.78$ d) $3.204 \div 0.0078 \div 0.007$

Don't forget to use the $\boxed{\times 10^x}$ ***key on your calculator.***

2. Calculate:

a) $(3.6 \times 10^4) \times (5.32 \times 10^7)$ b) $(3.71 \times 10^{-7}) \times (3.28 \times 10^{-3})$

c) $(3.1 \times 10^3)^2$ d) $(4.6 \times 10^{20}) \div (7.2 \times 10^6)$

e) $(5.28 \times 10^7) + (8.6 \times 10^7)$

Percentages

RULE 1: *Per cent means per hundred, i.e.* $1\% = \frac{1}{100}$

Thus $\quad 70\%$ *means* 70 out of 100

and can be written as a fraction $\quad \frac{70}{100}$ *or* $\left(\frac{7}{10}\right)$

or a decimal $\quad 0.7$

Centum is Latin for 100. While we don't speak Latin, we still use Latin in some terms like **per cent** and **century**.

Remember 100% *means* the whole amount.

We are often required to express percentages as fractions, or fractions as percentages.

A. Changing Percentages to Fractions

Remember $\quad 20\%$ *means* $\frac{20}{100}$

Thus

$$20\% = \frac{20}{100}$$

$$= \frac{1}{5}$$ *(when simplified by dividing the top and bottom by 20)*

EXAMPLE 1: Express the following as fractions.

a) 35% $\quad$ b) $7\frac{1}{2}\%$ $\quad$ c) 130%

Solution:

a) $$35\% = \frac{\overset{7}{\cancel{35}}}{\underset{20}{\cancel{100}}} = \frac{7}{20}$$

b) $$7\tfrac{1}{2}\% = \frac{7\frac{1}{2}}{100} = \frac{\overset{3}{\cancel{15}}}{\underset{40}{\cancel{200}}} = \frac{3}{40}$$

multiply top and bottom by 2

c) $$130\% = \frac{13\cancel{0}}{10\cancel{0}} = 1\tfrac{3}{10}$$

SOME USEFUL FRACTION/DECIMAL/PERCENTAGE RELATIONSHIPS

FRACTION	DECIMAL	PERCENTAGE
$\frac{1}{8}$	.125	12 ½%
$\frac{1}{4}$	.25	25%
$\frac{3}{8}$	.375	37½%
$\frac{1}{2}$	.5	50%
$\frac{5}{8}$	.625	62½%
$\frac{3}{4}$	.75	75%
$\frac{7}{8}$	.875	87½%

$\frac{1}{6}$	$.1\dot{6}$	16⅔%
$\frac{2}{6}\ \left(\frac{1}{3}\right)$	$.\dot{3}$	33⅓%
$\frac{3}{6}\ \left(\frac{1}{2}\right)$	.5	50%
$\frac{4}{6}\ \left(\frac{2}{3}\right)$	$.\dot{6}$	66⅔%
$\frac{5}{6}$	$.8\dot{3}$	83⅓%

OTHER USEFUL VALUES

$\frac{1}{16}$	.0625	6¼%
$\frac{1}{20}$	.05	5%
$\frac{1}{5}$	.2	20%

Exercise 4.2

1. Calculate:

a) 30% of 150 kg

b) 22% of $500

c) $33\frac{1}{3}$% of 60 m

d) 125% of 60

e) 45% of 6 hrs (in hours and minutes)

2. Express as a percentage.

a) 36 out of 90

b) 42 marks out of 60 marks

c) 45 minutes of 3 hrs

d) $5.40 of $3.60

e) 350 mL of 2 L

CHAPTER 5

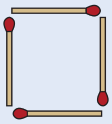
Algebra

What is Algebra?

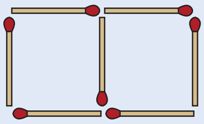
Algebra is a branch of mathematics which uses symbols (usually letters of the alphabet) to represent numbers, e.g. $E = mc^2$.

EXAMPLE 1:

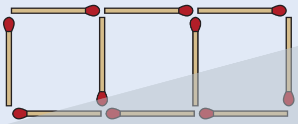
a) Complete the next two steps in the pattern and find a rule which fits this pattern.



1 square



2 squares

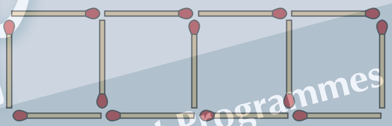


3 squares

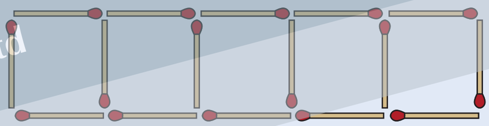
b) How many matches are there if there are 7 squares?
i.e. What is the value of m when $n = 7$?

Solution:

a)

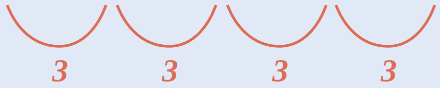


4 squares



5 squares

No. of squares (n)	1	2	3	4	5
No. of matches (m)	4	7	10	13	16



The number of matches is 3 times the no. of squares + 1.

$\therefore \quad m = 3n + 1$

b)

$$\begin{aligned} m &= 3n + 1 \\ &= 3 \times 7 + 1 \\ &= 21 + 1 \\ m &= 22 \end{aligned}$$

$\therefore$ The number of matches if there are 7 squares is 22.

A. The Distributive Law

RULE 1: *When expanding a bracket, the number next to the bracket must be multiplied through the whole bracket.*

thus $3(a + 2) = 3 \times a + 3 \times 2$

$= 3a + 6$

or $3(a + 2) = 3a + 6$

EXAMPLE 2: Expand:

a) $5(2a + 3b)$
b) $^{-}2(x + 2y)$

Solution:

a) $5(2a + 3b) = 5 \times 2a + 5 \times 3b$

$= 10a + 15b$

b) $^{-}2(x + 2y) = {}^{-}2 \times x + {}^{-}2 \times 2y$

$= {}^{-}2x - 4y$

EXAMPLE 3: Simplify the expression $5a + 2(3a - 4)$.

Solution:

Expand the brackets

Collect like terms

$$5a + \boxed{2(3a - 4)} = \boxed{5a + 6a} - 8$$

$$= 11a - 8$$

EXAMPLE 4: Expand $7x(2y - 4)$.

Solution:

$$7x(2y - 4) = 14xy - 28x$$

Ratio, Rates and Proportions

RATIO

Equivalent ratios are similar to equivalent fractions.

A **ratio** expresses the size of two quantities relative to each other. The ratio of two quantities indicates how many times one quantity is contained in another. For example, if the ratio of cars to trucks is 5 : 1, then there are 5 times as many cars as trucks.

Ratios are best expressed as two integers.

A. Expressing Ratios as Integers

When expressed as a common fraction

For example, the number of girls in a class is ⅓ the number of boys.

Then the ratio **girls : boys** can be expressed as ⅓ : 1
or by multiplying by 3, the ratio becomes 1 : 3.

When expressed as a decimal fraction

For example, the number of motorcycles to motor scooters is 2 : 0.7.

Convert the decimal to a whole number by multiplying by 10, then the ratio becomes 20 : 7.

EXAMPLE 1: If 10 people can make 35 bicycles in a day, how many bicycles can 27 people make?

Solution: This is a ratio problem.

people	:	*bicycles*
10	:	35
27	:	*b*

Write as equivalent fractions ensuring the pronumeral is on the top line on the left-hand side; then solve the equation.

$$\frac{b}{35} = \frac{27}{10}$$

$$\times 35, \quad b = \frac{27}{10} \times \frac{35}{1}$$

$$= \frac{945}{10}$$

$$= 94.5$$

∴ 27 people can make 94 complete bicycles in a day.

5. Divide the following in the given ratios.

a) 2000 in a ratio of 3 : 7

____

____

____

b) 3.6 in the ratio of 2 : 7

____

____

____

c) 15 300 in the ratio of 2 : 7

____

____

____

USING CALCULATORS FOR FRACTIONS

Most calculators have a **fraction button**, it usually looks like [$a\frac{b}{c}$].

This function allows you to type in mixed numbers and fractions.

To input ½ Press 1 [$a\frac{b}{c}$] 2 [=] ½

To input $4\frac{2}{5}$ Press 4 [$a\frac{b}{c}$] 2 [$a\frac{b}{c}$] 5 [=] $4\frac{2}{5}$

If your calculator does not have a fraction button:

To input ½ Press 1 [÷] 2 [=] .5

To input $4\frac{2}{5}$ Press 4 [+] 2 [÷] 5 [=] 4.4

Note: *The answers will be in decimal form.*

Exercise 6.3

1. Calculate the following rates in the units given.

   a) $10 for 1.7 kg ($/kg)

   b) 670 km using 45 L (km/L)

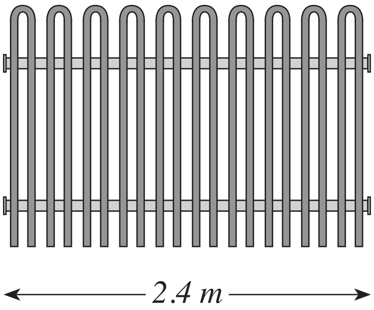
   c) 1 km in 1 minute (metres/second)

2. Jai's car uses 55 L of fuel to travel 500 km, while Lilly's car uses 30 L of fuel to travel 270 km. Which car has the better consumption rate?

3. Which is the better buy?

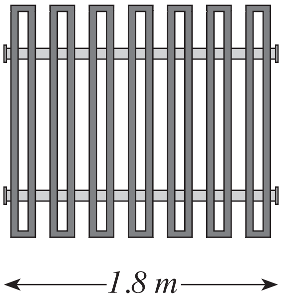
Cool Pool Fencing

$^{\$}$90



Safety Pool Fencing

$^{\$}$65



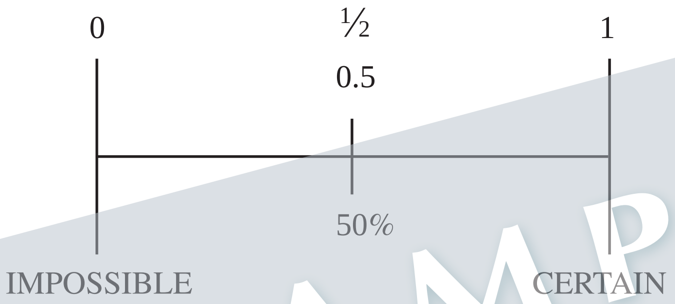
CHAPTER 7

Probability and Statistics

PROBABILITY

RULE 1: *Probability is the chance an event may occur. It is expressed as a value from 0 → 1 either in fraction, decimal or percentage form.*

A. Probability Continuum



RULE 2: *The probability of events occurring is equal to the number of successful outcomes divided by the total number of outcomes.*

$$P(E) = \frac{Number\ of\ Successes}{Total\ Number\ of\ Outcomes}$$

EXAMPLE 1: There are 6 red marbles, 2 blue marbles and 12 green marbles. What is the probability of choosing a red marble?

<u>*Solution:*</u>

$$P(E) = \frac{Number\ of\ Successes}{Total\ Number\ of\ Outcomes}$$

$$= \frac{6}{20} \quad \begin{array}{l} \longleftarrow no.\ of\ red\ marbles \\ \longleftarrow total\ no.\ of\ marbles \end{array}$$

$$= \frac{3}{10} \quad or \quad 0.3 \quad or \quad 30\%$$

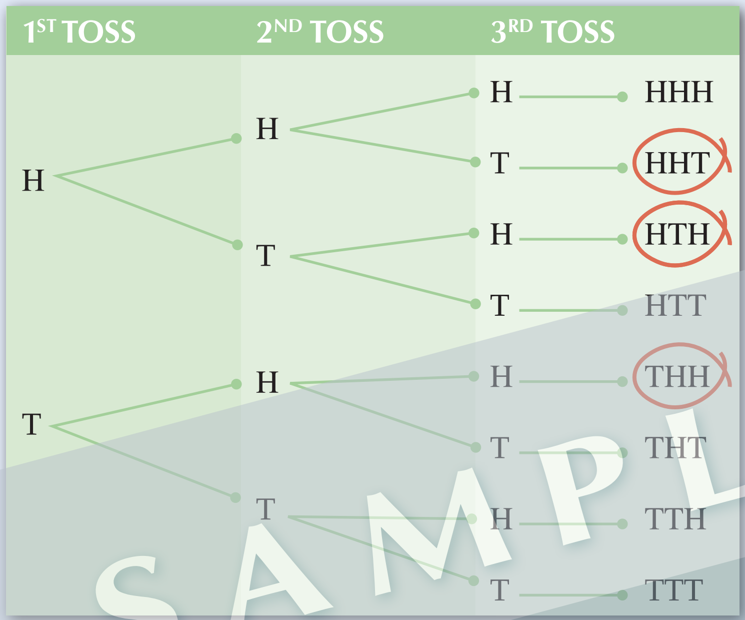
∴ The probability of choosing a red marble is 30%.

C. Tree Diagrams

Tree diagrams are a way of visualising all possible outcomes when 2 or more different events occur.

EXAMPLE 3: What is the probability of getting 2 heads and a tail when tossing a coin 3 times?

Solution:



$$\therefore\ P(2\ Heads\ and\ a\ Tail) = \frac{3}{8}$$

Exercise 7.1

1. There are 4 red, 3 blue, 6 green and 7 purple marbles.

a) What is the probability of getting a blue?

b) What is the probability of getting a green?

c) What is the probability of getting a blue or red?

d) What is the probability of *not* getting a purple?

E. Stem and Leaf Graphs

A quick way to get a visual impression of the spread of a set of statistical data is to use a stem and leaf graph. This is done by breaking the numbers down according to their place values and arranging them in ascending order.

EXAMPLE 6: Using the following data, draw a stem and leaf graph of the marks (out of 60) one Year 7 class achieved in a Science Test.

21, 37, 46, 22, 7, 31, 52, 27, 29, 41, 36, 27, 33, 35, 13, 36, 17,
14, 31, 41, 33, 53, 24, 44, 19, 26, 8, 22, 18, 35

Solution:

tens	*units*
0	
1	
2	1
3	
4	
5	

Look through the numbers and list the values of the ***tens column*** *of each number to the left of the line.*

Then go through placing the values of the ***units column*** *of each number to the right of the line.*

∴ *21 would then be placed with '2' in the* ***tens column*** *and '1' in the* ***units column****.*

Continue going through the list, placing the ***tens column*** figure and the ***units column*** figure (for each of the numbers) into the correct row.

tens	*units*
0	7 8
1	3 7 4 9 8
2	1 2 7 9 7 4 6 2
3	7 1 6 3 5 6 1 3 5
4	6 1 1 4
5	2 3

It is a good idea to double-check you've recorded all the data, i.e. 30 numbers in total.

Statistical data must have a title and a key

Year 7 Marks in Science

Key 1 | 3 = 13

tens	*units*
0	7 8
1	3 4 7 8 9
2	1 2 2 4 6 7 7 **9**
3	**1** 1 3 3 5 5 6 6 7
4	1 1 4 6
5	2 3

Place all numbers in the ***units column*** *in ascending order.*

16th number *15th number*

NOTE

The **key** indicates the place value of the 'stem' and 'leaf.'

Key 2 | 7 = 270

Stem is ***100s*** *Leaf is* ***10s***

The median is easily found from the graph by counting to the middle of the distribution. There are 30 numbers, so the median will be half-way between the 15th and 16th numbers.

$$\therefore\ Median = \frac{29+31}{2}$$
$$= 30$$

F. Box and Whisker Plot

The box and whisker plot is another way to visually represent data. It shows valuable information about the distribution. This method of graphing data makes use of the **median**, but also the **quartiles**. To find the quartiles, the data are arranged in order and divided into quarters. **This is first done by finding the median (i.e. the 2nd quartile).**

✻ **EXAMPLE 7:** Let's use the same data as in Example 6:

21, 37, 46, 22, 7, 31, 52, 27, 29, 41, 36, 27, 33, 35, 13, 36, 17, 14, 31, 41, 33, 53, 24, 44, 19, 26, 8, 22, 18, 35

Solution:

Step 1: Arrange the data in order of size.

7, 8, 13, 14, 17, 18, 19, 21, 22, 22, 24, 26, 27, 27, 29, 31, 31, 33, 33, 35, 35, 36, 36, 37, 41, 41, 44, 46, 52, 53

Step 2: Find the median. $\therefore$ *Median*

$$= \frac{29 + 31}{2}$$

$$= 30$$

Step 3: Firstly, find the middle number of the bottom half of the data (1st quartile), and then the middle number of the top half of the data (3rd quartile).

An easy way to find the middle if you have 15 numbers is:

$$\frac{15+1}{2} = 8$$

$\therefore$ *the 8th number*

There are 15 numbers below the median, thus the middle number would be the 8th number in the bottom half of the data.

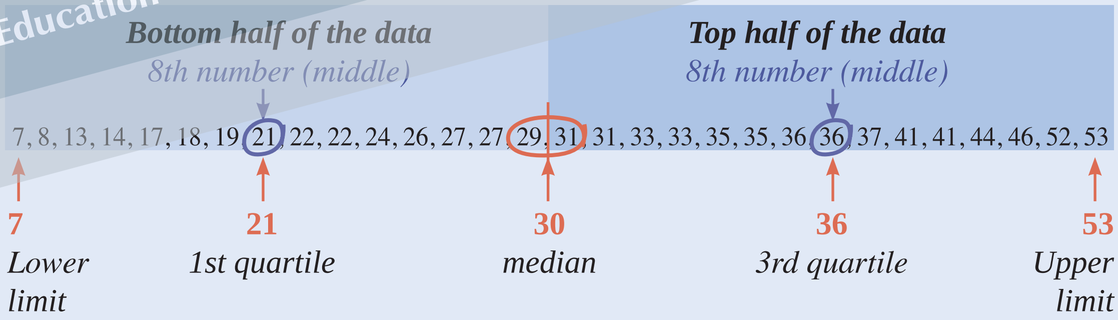
$\therefore$ 1st quartile = 21

The same theory applies for the top half of the data.

$\therefore$ 3rd quartile = 36

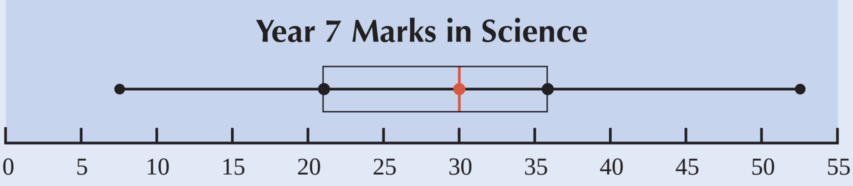
Step 4: Note the lower limit (7) and the upper limit (53).

The final results should be as follows:



The five numbers (7, 21, 30, 36, 53) are called the ***five figure summary*** of the data.

Step 5: The box and whisker plot is constructed using the ***five figure summary*** as follows:

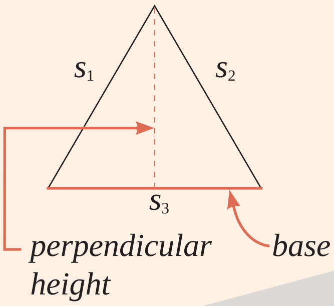
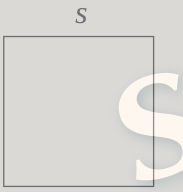
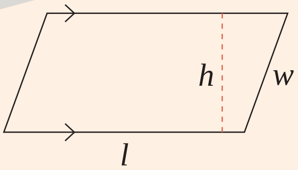
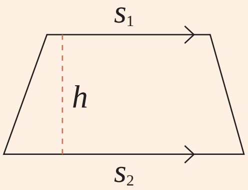
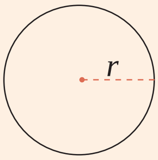


Every box and whisker plot must have a scale →

Measurement

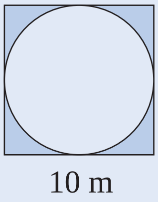
Perimeter and Area

RULE 1: *Formulae for calculating **perimeter** and **area** of regular shapes.*

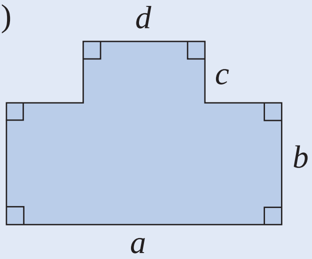
FIGURE	PERIMETER	AREA
Triangle 	$P = s_1 + s_2 + s_3$	$A = \frac{1}{2} \times$ *base* $\times$ *height*
Square 	$P = 4s$	$A = s^2$
Rectangle 	$P = 2l + 2w$	$A = l \times w$
Parallelogram 	$P = 2l + 2w$	$A = l \times h$
Trapezium 	$P =$ *sum of the sides*	$A = \frac{s_1 + s_2}{2} \times h$
Circle 	$C = 2\pi r$ or $C = \pi d$	$A = \pi r^2$

EXAMPLE 6: Find the area of the following shaded figures:

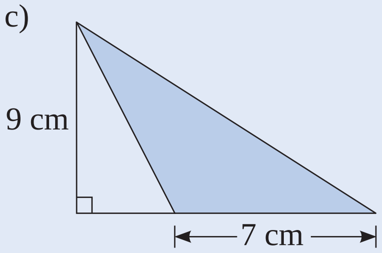
a)



b)



c)



Solution:

a) $$\begin{aligned} \textit{Area} &= \textit{Area of square} - \textit{Area of circle} \\ &= s^2 - \pi r^2 \\ &= 10^2 - \pi 5^2 \\ &= 100 - 78.54 \\ &= 21.46 \text{ m}^2 \end{aligned}$$

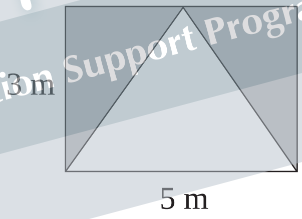
b) $$\begin{aligned} \textit{Area} &= \textit{Area of rectangle 1} + \textit{Area of rectangle 2} \\ &= ab + cd \text{ square units} \end{aligned}$$

c) $$\begin{aligned} \textit{Area of triangle} &= \tfrac{1}{2} \times \textit{base} \times \textit{height} \\ &= \tfrac{1}{2} \times 7 \times 9 \\ &= 31.5 \text{ cm}^2 \end{aligned}$$

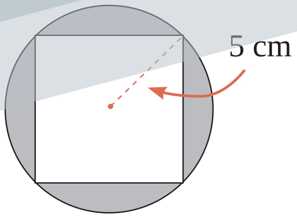
Exercise 8.3

1. Find the area of the shaded figures:

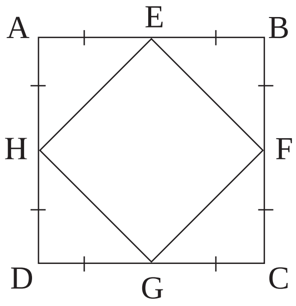
a)



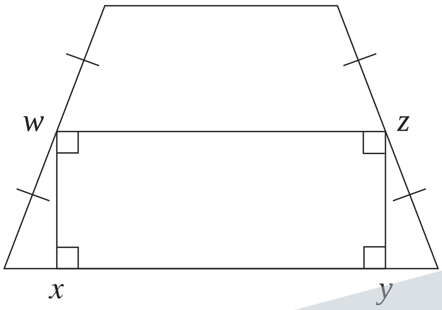
b)



2. Prove that the area of A B C D is twice the area of E F G H.



3. The area of the trapezium is 30 cm^2. What is the area of the rectangle?



4. A circle has an area of 380.13 m^2. What is the radius? (Give your answer to the nearest whole number.)

5. A farmer has 60 m of fencing. He is unsure if he can enclose more area by making a square paddock or a rectangular one 20 m × 10 m. Which shape should he choose? By how many square metres is it larger?

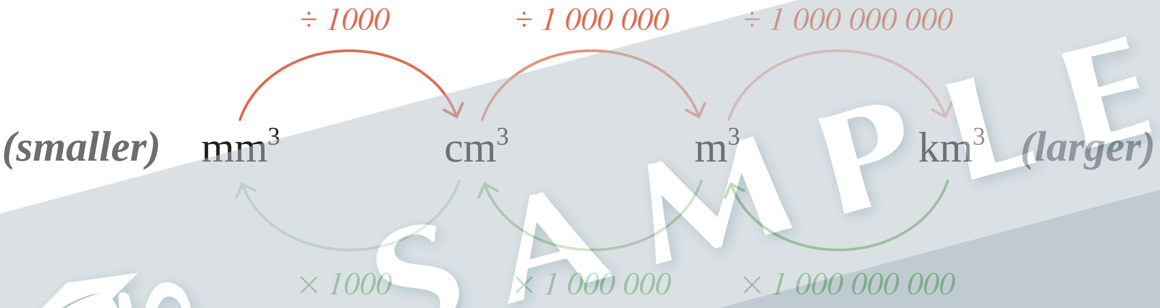
D. Units of Volume

Volume is the number of **cubic units** a three-dimensional space occupies.

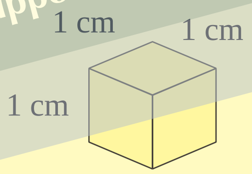
RULE 4: *Formulae for calculating **volume** of common figures.*

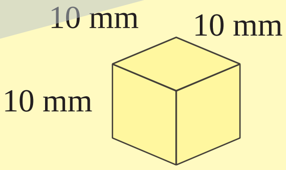
Cube	$V = s^3$
Prism	$V = \text{Area of base} \times \text{height}$
Cone or Pyramid	$V = \frac{1}{3} \times \text{Area of base} \times \text{height}$

Volume Conversions



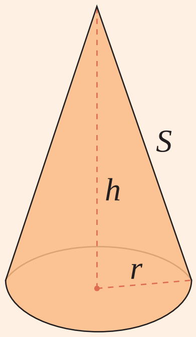
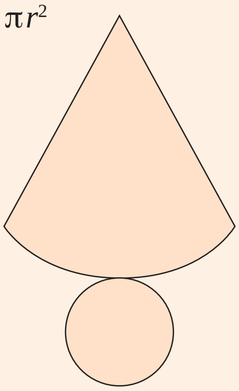
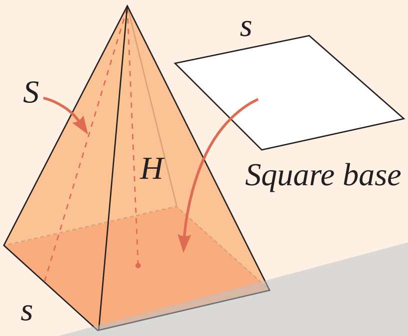
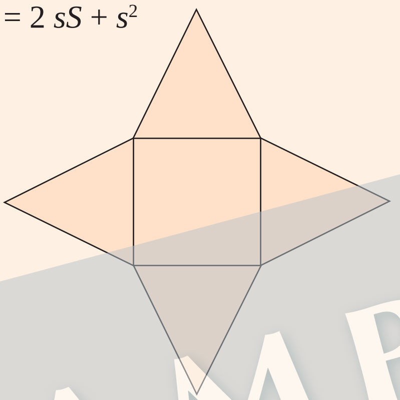
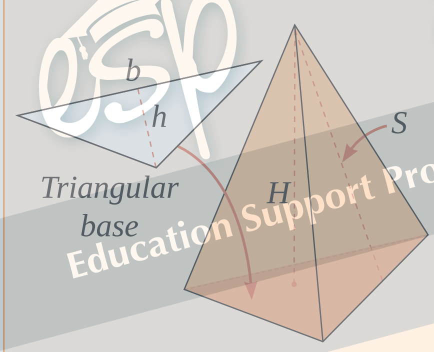
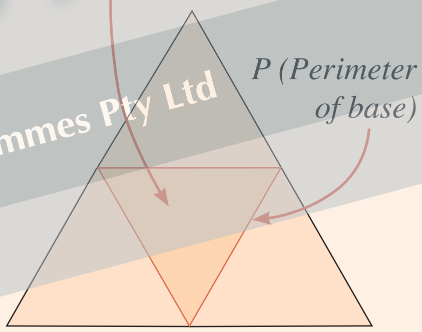
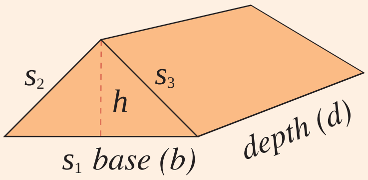
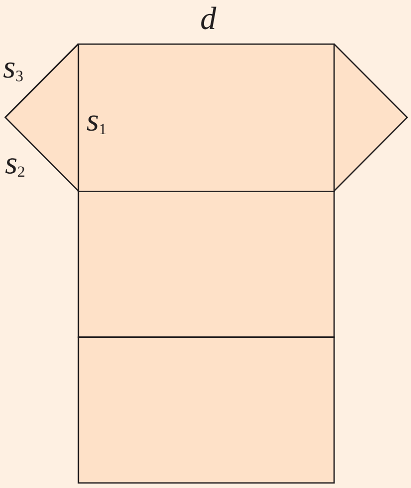
Using different units of measure to describe the same volume





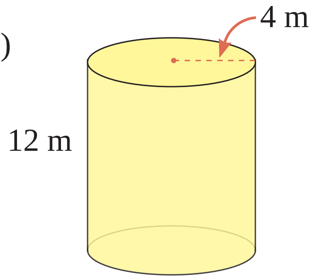
$V = 1\ \text{cm}^3$ $\quad$ $V = 1000\ \text{mm}^3$

$$\therefore 1\ \text{cm}^3 = 1000\ \text{mm}^3$$

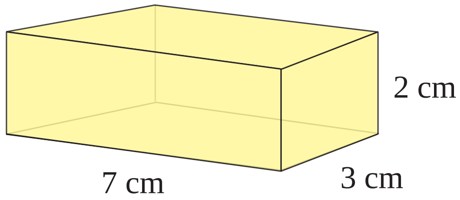
FIGURE	SURFACE AREA	VOLUME
Cone  	$SA = \pi r S + \pi r^2$ 	$V = \frac{1}{3}\pi r^2 h$ (⅓× *volume of a cylinder)*
Square-based Pyramid  	$SA = 2\ sS + s^2$ 	$V = \frac{1}{3} s^2 H$ (⅓× *volume of a rectangular prism)*
Triangular-based Pyramid  	$SA = \underbrace{\text{Area of base}} + \frac{1}{2} PS$  	$V = \frac{1}{3}(\frac{1}{2}\, b\, h\, H)$ (⅓× *volume of a triangular prism)*
Triangular Prism  	$SA = (s_1 + s_2 + s_3)\ d + s_1 h$  	$V = \frac{1}{2}\, b\, h\, d$

2. Find the volumes of the following figures:

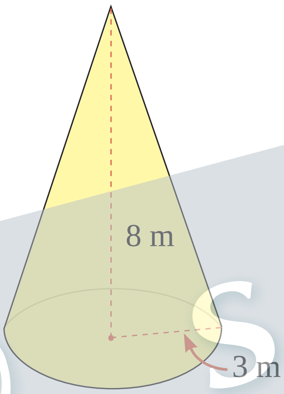
a)



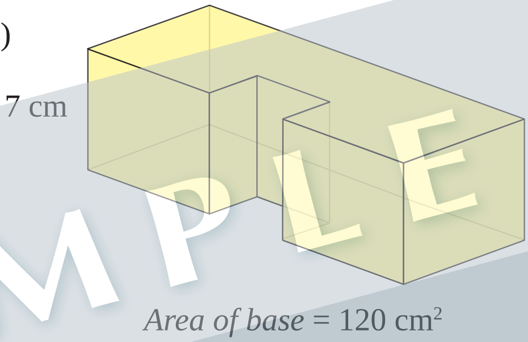
b)



c)



d)



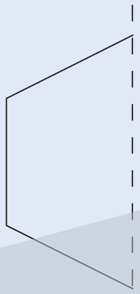
3. Find the volume of a can 6 cm in diameter and 12 cm high.

Geometry

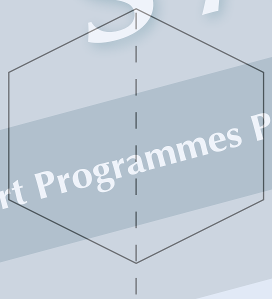
Geometry is the branch of mathematics that deduces the properties of figures in space.

A. Symmetry

EXAMPLE 1: Complete the following shape, given that it is symmetrical about the dotted line.



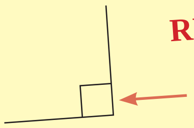
Solution:



The line of symmetry acts like a mirror reflecting an image.

∴ The shape is a hexagon.

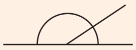
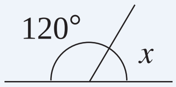
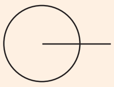
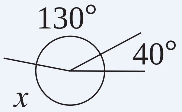
REMEMBER



This is the symbol for a right angle (90°).

B. Angles from Straight Lines

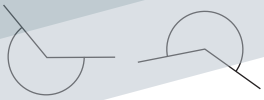
RULE 1: *The following rules apply to **straight** lines.*

SYMBOL	MEANING	✱ EXAMPLE 2
	Angles on a straight line add up to 180°.	 $x = 180 - 120$ $= 60°$
	A revolution is 360°.	 $x = 360 - 130 - 40$ $= 190°$
	Vertically opposite angles have the same value.	 $x = 50°$

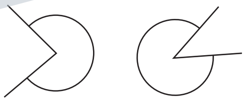
C. Types of Angles

Thre are three types of angles:

Acute angles are angles less than 90°.



Obtuse angles are angles between 90° and 180°.



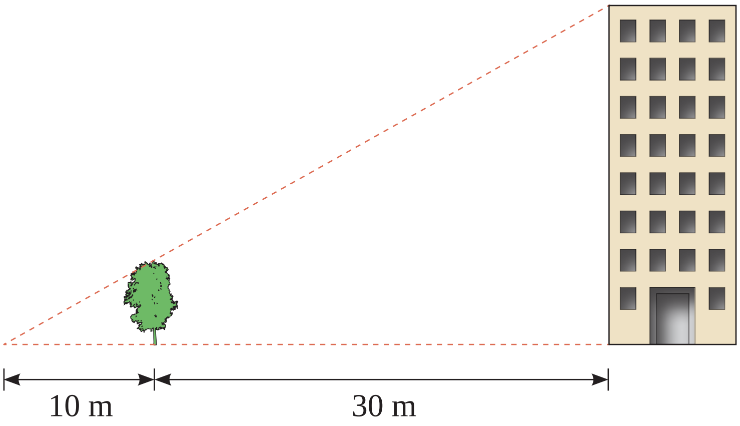
Reflex angles are angles between 180° and 360°.

NOTE

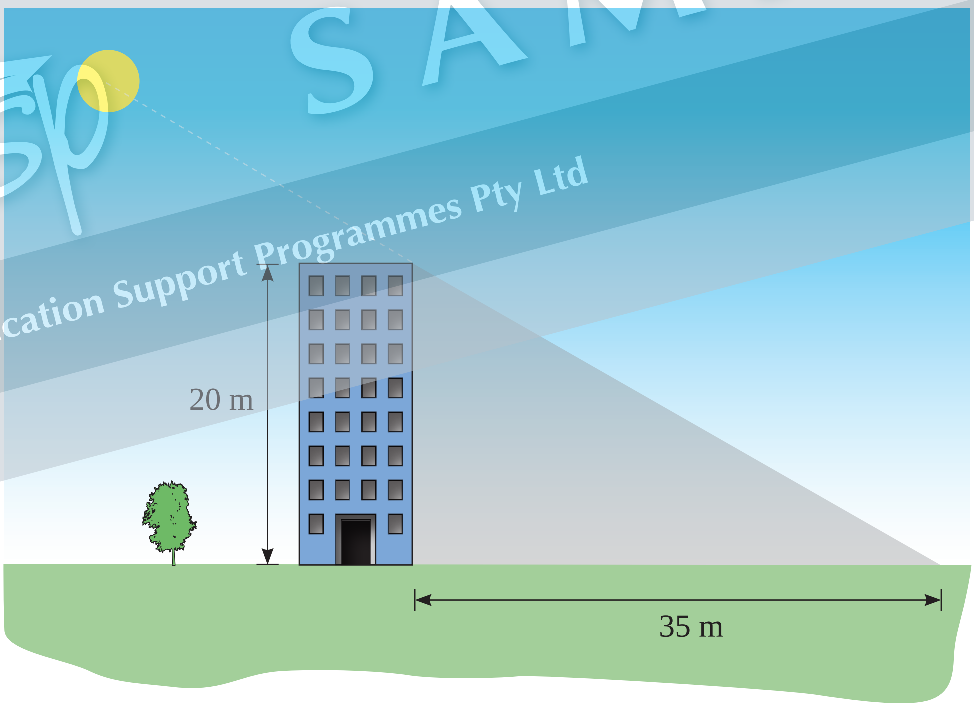
180° is a straight line angle, neither acute or obtuse.

360° is a revolution and is not a reflex angle.

3. Find the height of the building if the height of the tree is 4 metres.



4. If Carlos is 1 m 70 cm tall, what is the furthest distance he can stand from the building and still be completely shaded from the sun?



G. Polygons

Polygon comes from a Greek word meaning ‘many angled’ and refers to all enclosed figures. Triangles, rectangles and squares are examples of polygons with which you are already familiar.

Polygons are named after their angles and the Greek names for numbers are used:

5 — penta
6 — hexa
7 — hepta
8 — octa
9 — nona
10 — deca
11 — undeca
12 — dodeca

NOTE
Greek names are often used in mathematics because Western mathematics is based on the work of Greek mathematicians.

Thus a pentagon has 5 sides, and dodecagon has 12 sides.

The Greeks had a system (based on their number system) which named figures with any number of angles. A figure with 572 angles would be a:

`penta hecta heptaconta kai digon`

↑ ↑ ↑ ↑ ↑

`5 00 70 and 2`

Fortunately you will rarely need more than a 12-sided figure in any mathematics you will ever do! ***If you are fascinated by this, search the internet.***

COMPARING SIDES AND ANGLES OF A POLYGON

NAME	SIDES	TOTAL DEGREES OF ANGLES
triangle	3	180°
rectangle	4	360°
pentagon	5	540°
hexagon	6	720°
heptagon	7	900°

NOTE
For every additional side, add 180°.

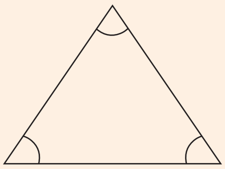
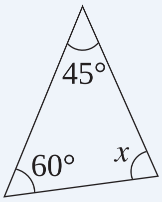
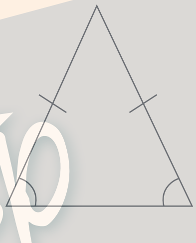
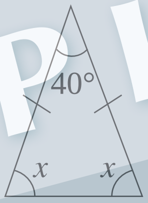
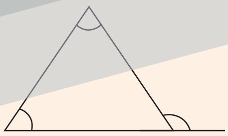
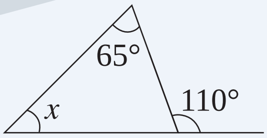
Exercise 9.4

1. Make up a rule which links the number of sides to the number of degrees. (Check the answer with B.O.B.!) *It is a handy rule and may help you answer a NAPLAN question*.

H. Angles from Polygons

Triangles

RULE 6: *The following rules apply to **triangles**.*

SYMBOL	MEANING	✻ EXAMPLE 4
	Angles in a triangle add up to 180°.	 $x = 180 - 60 - 45$ $= 75^\circ$
	The angles opposite the equal sides of an isosceles triangle are equal.	 $2x = 180 - 40$ $= 140$ $x = 70^\circ$
	The exterior angle of a triangle equals the sum of the two interior opposites.	 $x + 65 = 110$ $= 110 - 65$ $x = 45^\circ$

Co-ordinate Geometry

A. Co-ordinates

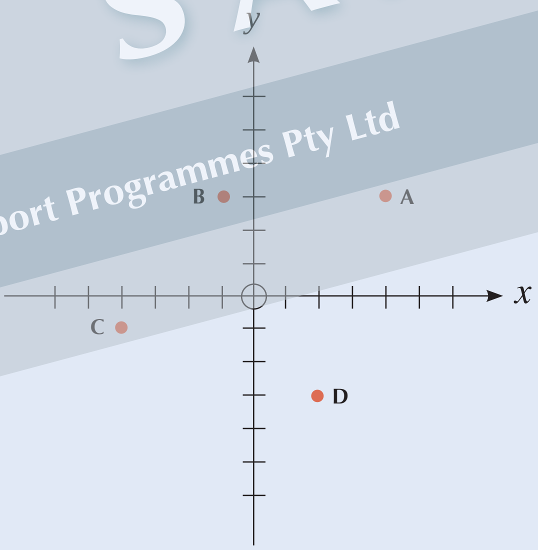
RULE 1: *To remember which orientation the x-axis and y-axis are, use the following:*

- *x is a 'cross' (and goes across).*
- *y points to the sky.*

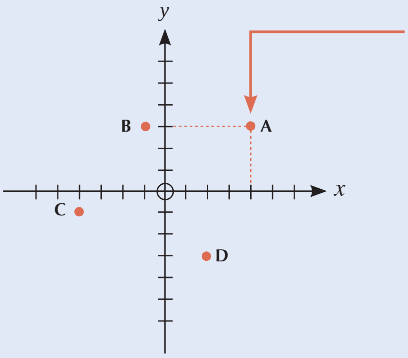
∴ x is the horizontal axis and y is the vertical axis.

RULE 2: *When describing the position of a point, the x co-ordinate is always given before the y co-ordinate.*

EXAMPLE 1: Give the co-ordinates of A, B, C and D from the graph below.



Solution:



*A has an **x** co-ordinate of 4 and **y** co-ordinate of 3.*

∴	A	(4, 3)
	B	$(^-1, 3)$
	C	$(^-4, ^-1)$
	D	$(2, ^-3)$

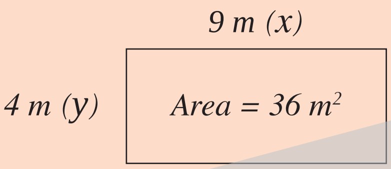
Graphs

Interpreting Graphs

The NAPLAN Test asks students to interpret a wide variety of graphs. The questions focus mainly on testing your understanding of the relationship between the horizontal and vertical axes. This chapter will give you strategies to help you solve these problems.

RULE 1: *When graphing, remember:*

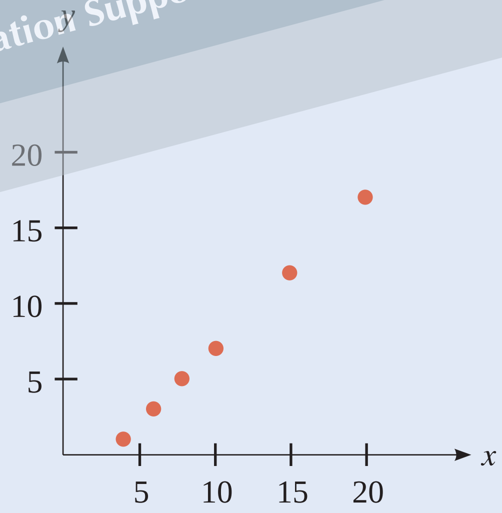
- *The **horizontal axis** (x) is used to measure the **independent variable** — these values are chosen by the creator of the graph.*
- *The **vertical axis** (y) is the **dependent variable** and is the value which is derived from an equation or experiment.*



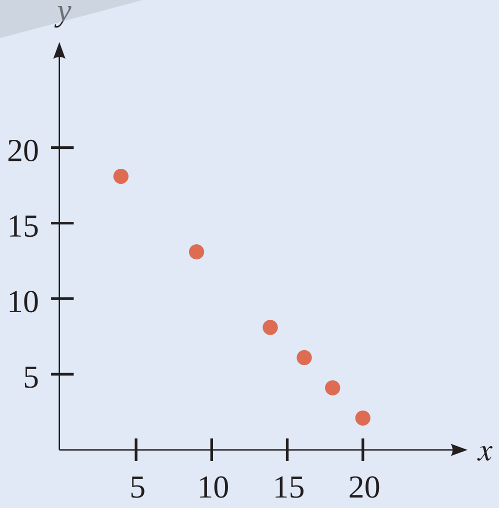
*For example, if a rectangle has an area of 36 m² and you choose a length of 9 m (the **independent variable**) [horizontal axis (x)], then the width is 4 m (the **dependent variable**) [vertical axis (y)].*

EXAMPLE 1: Which graph reflects the following set of values?

x	4	6	8	10	15	20
y	1	3	5	7	12	17



○ ***Graph A***



○ ***Graph B***

Solution: By examining the table of values it can be seen that as *x* increases *y* increases. This is true of Graph A only, thus Graph A is correct.

Exercise 12.1

1. Find the next two numbers in the following sequences:

a) $1, 2, 4, 7, 11,$ ____, ____

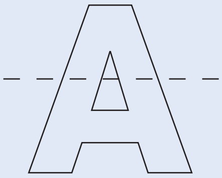
b) $1, {}^-1, 2, 0, 3,$ ____, ____

c) $1, {}^-4, 9, {}^-16, 25,$ ____, ____

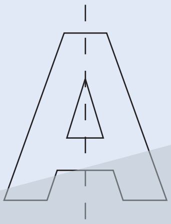
d) $2, 1, 4, 3, 6,$ ____, ____

A. Lines of Symmetry

EXAMPLE 2: Which of the two figures represents a line of symmetry?



○ *Figure A*



○ *Figure B*

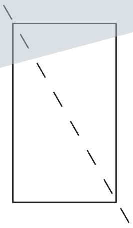
Solution: Take each figure and imagine folding it along the dotted line. If the two halves fold exactly on top of each other then the line is a line of symmetry.

$\therefore$ The line in Figure B is a line of symmetry.

Exercise 12.2

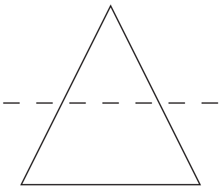
1. Which of the following are lines of symmetry?

a)



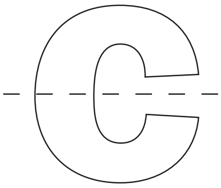
○

b)



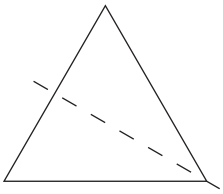
○

c)



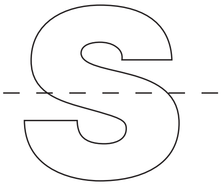
○

d)



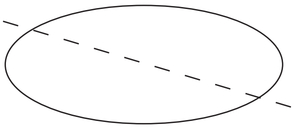
○

e)



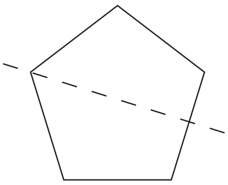
○

f)



○

g)



○

NAPLAN-style Numeracy Tests

Remember to use a 2B pencil only.

TEST ONE

(with calculator)

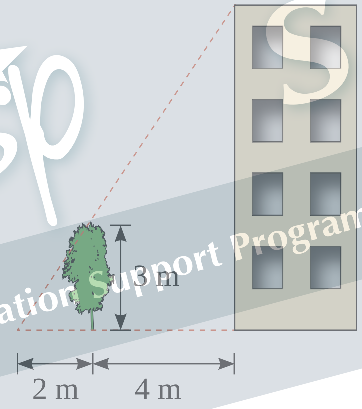
1 If $a = 4$, the value of $2a^2$ is?

64	8	32	16
○	○	○	○

2 From a 6 m length of timber a carpenter cuts 3 lengths — 720 mm, 2400 mm and 2650 mm. What length remains?

320 mm	340 mm	.5 mm	230 mm
○	○	○	○

3



The height of the building is

6 m	9 m	3 m	10 m
○	○	○	○

4 If one Australian dollar will buy 0.65 of a Euro, how many Euros can be bought with $150?

97.5	230.8	150	149.35
○	○	○	○

5 Isabel spent the following time on her homework during the week: 50 minutes, 75 minutes, 80 minutes, 65 minutes and 45 minutes. What is the average time she spent on her homework each night?

315 mins	63 mins	54 mins	50 mins
○	○	○	○

6 Dad gives his 4 children \$30 to share. Marian spends ⅔ of her share. How much does she have left?

\$5	\$7.50	\$3.33	\$2.50
○	○	○	○

7 A garment marked at \$56 was purchased for \$42. The percentage discount was

20%	25%	33 ⅓%	75%
○	○	○	○

8 A common recipe for cupcakes has the mass of butter : sugar : flour in the ratio of 4 : 6 : 8. If I use 300 g of sugar, how much flour should I use?

200 g	300 g	400 g	500 g
○	○	○	○

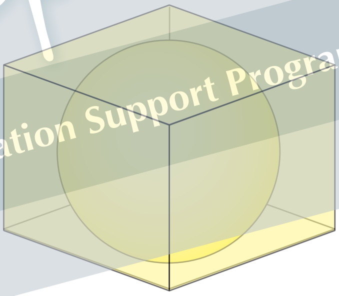
9 The grades in a test are calculated using the table:

A	B	C	D
$\geqslant 90\%$	$\geqslant 70\%$	$\geqslant 50\%$	$\geqslant 25\%$

What grade is achieved by a student who scored 32 marks out of 40 marks?

A	B	C	D
○	○	○	○

10



A solid spherical china ornament is packed in a wooden box just large enough to hold the sphere. If the box is a cube with an internal measurement of 5 cm, what is the best answer for the volume of the empty space in the box?

Volume of a cube $V = s^3$

Volume of a sphere $V = \frac{4}{3}\pi r^3$

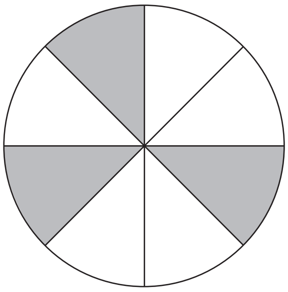
125 cm^3	59.6 cm^3	98.8 cm^3	65.4 cm^3
○	○	○	○

11 The price of petrol varies during the week. I have a car whose tank holds 60 L and has an economy rate of 7 km/L. How much further can I travel if I spend \$50 on petrol bought on Tuesday for \$1.03/litre than \$50 spent on Friday for \$1.18/litre (answer to the nearest km)?

40	43	47	44
○	○	○	○

(without calculator)

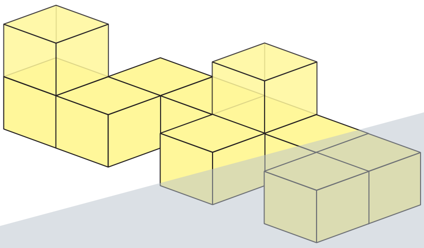
1



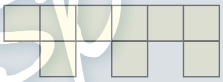
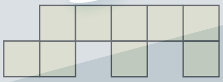
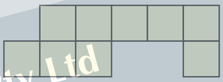
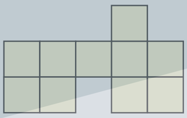
What fraction of the whole is represented by the shaded area?

$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{2}{3}$
○	○	○	○

2



Which of these shows the top view?

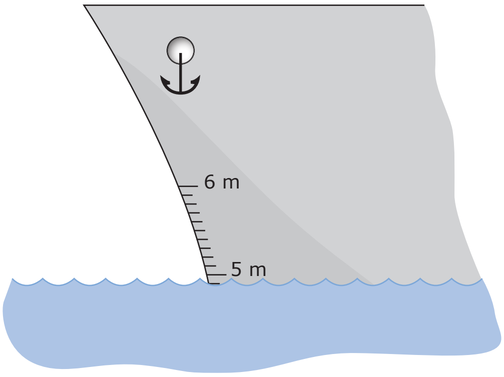





○	○	○	○

3 When $a = 2$ and $b = {}^{-}3$, find the value of $a^3 - b^2$.

15	$^{-}3$	17	$^{-}1$
○	○	○	○

4 How deep is the ship in the water?



5.1 m	49 m	4.9 m	51 m
○	○	○	○

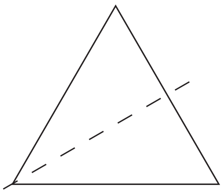
11 If $a = 4$, what is the value of $\dfrac{5a}{3a - 2}$?

20	2	10	$1\frac{2}{3}$
○	○	○	○

12 What is the best estimate of $21 \times 34 - 48 + 97$?

700	664	750	714
○	○	○	○

13 Which of the following represents a line of symmetry?



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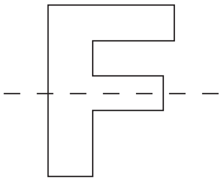
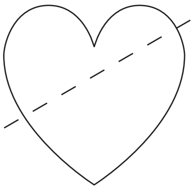
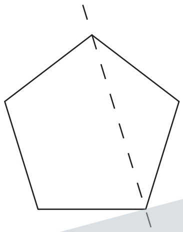


○

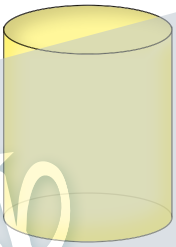


○

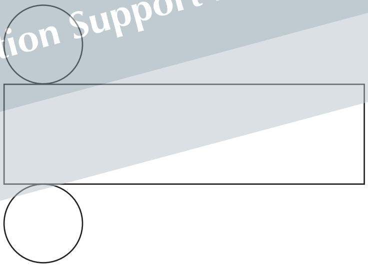


○

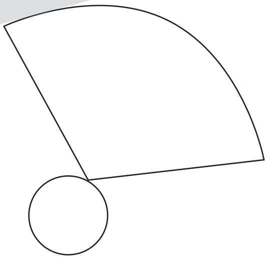
14 This is an open cylinder.



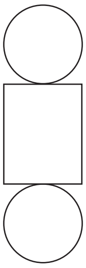
Which diagram represents a net of an open cylinder?



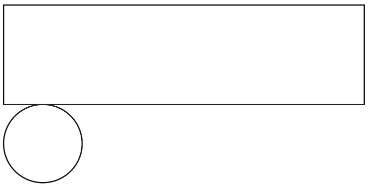
○ ***Diagram A***



○ ***Diagram B***



○ ***Diagram C***



○ ***Diagram D***